A Short Summary of the Mathematics of Calculus

*Everyday Calculus* is designed to gently introduce you to calculus by using everyday experiences, like drinking coffee, to reveal the hidden calculus all around you. This supplement is designed to complement the book by summarizing the mathematics of calculus discussed in the book. The exposition in this supplement is more mathematical (since that is the focus of this document). So, I will often refer to specific passages in the book that provide a more intuitive viewpoint of the math being discussed to reinforce what you learn from this supplement. I will color those references in blue text to help you spot them.

If you've stumbled upon this document without reading *Everyday Calculus* (or ever having studied calculus), I highly suggest reading *Everyday Calculus* first and then coming back to this supplement. The book will give you a good introduction to the ideas behind calculus, how they originated, how they're applied, and where they can be found in everyday life. You'll also learn some of the mathematics of calculus. Then, when you come back to this supplement, you'll see that math all in one place and in somewhat more detail. A quick preview of what's in here—the four pillars of Calculus I:

- Functions
- Limits
- Differentiation
- Integration

(Calculus II and onward add pillars, if you will; for example, infinite series are a pillar of Calculus II.) As we'll discuss later in this summary, derivatives and integrals are defined in terms of limits, and in calculus we almost always take limits of functions. So, these pillars are not independent of each other.

As always, please feel free to contact me with any comments, suggestions, or questions.

Sincerely,

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1 Functions

Nearly everything done in calculus is done to functions; for example, we find limits of functions, we differentiate functions, and we integrate functions. So let’s begin this lightning review of the math of calculus with a short discussion of functions.

**Definition 1: Functions.** Suppose the value of a variable quantity $y$ depends solely on the value of another variable quantity $x$. This relationship is called a function if for each input $x$ there is exactly one output $y$. We then write

$$y = f(x)$$

and say that “$y$ is a function of $x$.” We call $f$ the function, $x$ the independent variable, and $y$ the dependent variable. A particular $x$-value is called an input, and its associated particular $y$-value an output. The set of all inputs is called the domain of $f$; the set of all outputs is called the range of $f$.

(This definition is discussed on page 119 of *Everyday Calculus.*)

**EXAMPLE 1** The following relationships between $x$ and $y$ define functions.

- $y = x^2$
- $2x + y = 4$
- $y = x^3 - 4x + 1$

(The second can be solved for $y$ to yield $y = -2x + 4$.)

Each equation above satisfies the “for each input $x$ there is exactly one output $y$” requirement of Definition 1. Let’s contrast those examples with one that doesn’t define a function:

$$x^2 + y^2 = 1.$$ 

To see why this doesn’t define a function, suppose $x = 1$. Then $y^2 = 1$. This yields the two outputs $y = -1$ and $y = 1$ (more compactly, $y = \pm 1$ ("plus or minus 1").

Determining whether an equation involving $x$ and $y$ defines a function is easiest to do from the graph of the equation. By this we mean the plot of points $(x, y)$ in the plane that satisfy the equation. The “for each input $x$ there is exactly one output $y$” requirement of Definition 1 then becomes:

**Theorem 1: The Vertical Line Test.** The graph of an equation of two variables $x$ and $y$ defines $y$ as a function of $x$ if and only if every vertical line intersects the graph at most once.
(“If and only if” is used in math like “vice versa” is in English. So, the Theorem says that if every vertical line intersects the graph at most once then that graph defines a function, and vice versa (if the graph is that of a function then every vertical line intersects the graph at most once.) Appendix A in Everyday Calculus discusses the vertical line test, and illustrates it with Figure A.2.

The functions discussed in Example 1 are all particular kinds of polynomials, functions of the form

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0, \]

where the \( a_0, a_1, a_2, \ldots, a_n \) are real numbers, \( n \) is a non-negative integer (i.e., \( n = 0, 1, \ldots \)), and we suppose \( a_n \neq 0 \). This family of functions shows up often in calculus, as do the following other families of functions discussed in Appendix A of Everyday Calculus:

- Trigonometric functions (e.g., \( f(x) = \sin x \))
- Exponential functions (e.g., \( f(x) = 2^x \))
- Logarithmic functions (e.g., \( f(x) = \ln x \))

Functions do a great job of describing a variety of real-world phenomena, including temperature, your bank account balance, the spread of the common cold, and even your sleep cycle. These are among the many applications of functions discussed throughout Everyday Calculus (especially Chapter 1).

2 Limits

Here’s an intuitive definition of the limit that will suffice for this short introduction to the mathematics of calculus.

**Definition 2: Limits.** Let \( f \) be a function, and \( c \) and \( L \) real numbers. Suppose that as \( x \) gets closer to \( c \) from either side (but never reaches \( c \)), the \( y \)-values \( f(x) \) get closer to the same number \( L \). Then we will write

\[ \lim_{x \to c} f(x) = L, \]

read “the limit as \( x \) approaches \( c \) of \( f(x) \) is \( L \).” We may also shorten this to “as \( x \to c \) then \( f(x) \to L \).”

Chapter 2 of Everyday Calculus explores the limit concept in more detail. There I touch on the many subtleties of the concept, including:

- The limit may exist even if the function isn’t defined at \( x = c \) (see Figure 2.6 in Everyday Calculus and the discussion surrounding it).
- In some instances \( L = f(c) \), meaning the answer to the limit is the function’s \( y \)-value at \( x = c \). When this is true we say that \( f \) is **continuous at** \( c \); see equation (9) in Everyday Calculus for more information. Notably, however, not all limits have this property.
- The limit may not exist for a variety of reasons, including the function shooting off to infinity as \( x \to c \).
Limits are typically calculated by using algebra to first simplify the function, thereby transforming the limit problem into a simpler one, and then applying some “Limit Laws.” These Limit Laws are properties of limits that reduce limit problems to calculations of basic limits, like

\[ \lim_{x \to 1} x \]

(whose answer is 1). One can guess the value of a limit by constructing a limit table, a table of \( x \)- and \( y \)-values where \( x \) is close to \( c \); see Table 2.1 in *Everyday Calculus* for an example of a limit table, and how it’s used to estimate the value of a limit.

Finally, Definition 2 is only valid if \( f(x) \) approaches the same \( y \)-value \( L \) as \( x \) approaches \( c \) from both sides. But there are many instances in which \( f(x) \to L \) as \( x \) approaches \( c \) from values less than \( c \) (i.e., “from the left of \( c \)”), and \( f(x) \to M \neq L \) as \( x \) approaches \( c \) from the right of \( c \). This leads to the concept of one-sided limits; see equations (7)–(8) in *Everyday Calculus* for a discussion of one-sided limits.

### 3 Differentiation

A classic problem that influenced the development of calculus is the following: Given a function \( f(x) \), can one define the slope of the function at a certain point \((a, f(a))\) on the function? If the function is a linear function (a function of the form \( f(x) = mx + b \)) the answer is simple: yes. Simply define the slope of the function at any point on the function to be \( m \), since that’s the slope of the graph of the function. The real difficulty, then, lies in defining the slope of a nonlinear function. Mathematicians struggled with this problem for centuries because they tried to extend what already worked out—defining the slope of a linear function to be the slope of graph of the linear function (i.e., \( m \))—to the setting of nonlinear functions. In doing so they ran up against a basic fact: we need two points to calculate the slope of a line. Isaac Newton and others eventually solved the problem by introducing the derivative. Here’s the definition.

**Definition 3: The Derivative at a Point.** Let \( f \) be a function and suppose \( f(a) \) exists. Then the limit

\[ \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}, \] (1)

if it exists, is called the **derivative of \( f \) at \( a \)**, and will be denoted by \( f'(a) \). The function is then said to be **differentiable at** \( x = a \).

You’re probably wondering how this solves the “slope of the function at a point” problem. To see how, let’s first re-express the quantity we’re taking the limit of in equation (1):

\[ \frac{f(a + h) - f(a)}{h} = \frac{f(a + h) - f(a)}{(a + h) - a}. \] (2)

We’ll use Figure 1 on the next page to make sense of this.
Figure 1: The graph of a function $f(x)$, two points on this function—$(a, f(a))$ (in black) and $(a+h, f(a+h))$ (in blue)—and the secant line (dashed) passing through those two points. (I explain what the red line is below.)

Notice that the quantity $(a+h) - a$ in the denominator of (2) is the change in the $x$-value of $f$ from $x = a$ to $x = a + h$; let’s call that change $\Delta x$ (read “change in $x$”), so that $\Delta x = (a+h) - a$. That change in input produces a change in output ($y$-values) of $f(a+h) - f(a)$. Let’s call that change $\Delta y$, so that $\Delta y = f(a+h) - f(a)$. Then (2) becomes

$$f(a+h) - f(a) = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{\Delta y}{\Delta x}.$$ 

The quantity all the way on the right is the slope of the line connecting the points $(a, f(a))$ and $(a+h, f(a+h))$ in Figure 1 (the dashed line in the figure). That line is called the secant line. Returning to Definition 2, the limit in (1) then becomes

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x},$$

(since $\Delta x = (a+h) - a = h$, so that $h \to 0$ is equivalent to $\Delta x \to 0$). Thus, if the derivative exists, we have

$$f'(a) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}, \quad (3)$$

which tells us that the derivative is the limit of the slopes of the secant lines through $(a, f(a))$ and $(a+h, f(a+h))$ as the right-endpoint used to calculate those slopes gets closer to the left-endpoint (i.e., $\Delta x \to 0$). The resulting line—the red line in Figure 1—is called the tangent line, so named because if you zoom in to the point $(a, f(a))$ in Figure 1, the line is tangent to the graph of $f(x)$ at the point $(a, f(a))$. Figure 2.2 in Everyday Calculus illustrates the “secant to tangent” idea via an application to stock prices. The paperback edition of Everyday Calculus also contains a computer icon next to

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that Figure, indicating to the reader that I’ve created an interactive online graph to illustrate the Figure. In this case, you can visit

http://www.surroundedbymath.com/interactive−math/225−2/

to access an interactive applet of Figure 1 above (you can change \( f \) and the \( a \)-value in that applet and see the secant lines approaching the tangent line).

Our work has led us to a very important interpretation of the derivative: \( f'(a) \) is the slope of the line tangent to the graph of \( f \) at the point \((a, f(a))\). Said more succinctly, \( f'(a) \) is the slope of the tangent line. Getting back to the question posed at the start of this section, we can now define the slope of a function at a point \((a, f(a))\) on the graph of that function by \( f'(a) \). Great! But this only works if \( f'(a) \) exists. Since \( f'(a) \) is defined as a particular limit, we’re back to the questions of calculating limits and the existence of limits we touched on in the last section. This is one manifestation of the interdependence of the four pillars of calculus I alluded to on the first page of this document.

Thus far we’ve only defined the derivative at a specific point. If we calculated \( f'(a) \) for all \( a \)-values in the domain of \( f \) and then plotted all of the points \((a, f'(a))\) we’d get the derivative function \( f'(x) \). It’s defined in much the same way as (1):

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},
\]

except that now we think of \( x \) as a variable and not the fixed number we thought of \( a \) as. Since \( f'(x) \) is still defined as a limit, once again, we’re back to calculating limits. Here’s a quick example of how we’d calculated the derivative of a quadratic function.

▶ EXAMPLE 2  Find \( f'(x) \) for \( f(x) = x^2 \).

Solution. From (4), we have:

\[
f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} (2x + h) = 2x.
\]

NOTE: \( h \) was canceled from the numerator and denominator to obtain the equality in red. That required dividing both the numerator and denominator by \( h \), which we can only do if \( h \neq 0 \) (division by zero is not defined). But that’s okay, because if you go back to Definition 2, in calculating the limit as \( x \to c \) we never consider \( x = c \) (in the case of the present example, \( x = h \) and \( c = 0 \), so that \( x \to c \) becomes \( h \to 0 \)).

Below I’ve graphed portions of \( f(x) = x^2 \) (blue) and it’s derivative function \( f'(x) = 2x \) (red). Notice that the slopes of the tangent lines to the graph of \( f \) are the \( y \)-values \( f'(a) \) on the graph of \( f' \). In particular, at \( x = 0 \) the tangent line to \( f \) is horizontal (zero slope), and indeed, \( f'(0) = 0 \). Moreover, for \( x > 0 \) the tangent lines to \( f \) have positive slopes (and indeed \( f'(x) > 0 \) for \( x > 0 \)), and for \( x < 0 \) the tangent lines to \( f \) have negative slopes (and indeed \( f'(x) < 0 \) for \( x < 0 \)).

3.1 Differentiation Rules

Calculating the derivative using (4) every time can get quite cumbersome. Luckily, just like the Limit Laws we discussed, there are “derivative laws,” which we call differentiation rules. These are
Figure 2: Portions of the graphs of \( f(x) = x^2 \) and \( f'(x) = 2x \).

easiest to state in Leibniz notation, so named for the co-inventor of Calculus, Gottfried Leibniz. This notation emerged as a straightforward extension of the right-hand side of (3). In this notation,

\[
f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}.
\]

(The new part is “\( \frac{dy}{dx} \).”) We think of this as an “infinitesimal change in \( y \)” divided by an “infinitesimal change in \( x \).” The notation is also sometimes used to denote “the derivative of \( f(x) \):

\[
\frac{d}{dx}[f(x)].
\]

Okay, on to the differentiation rules:

1. \( \frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] \)

2. \( \frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)] \) for any real number \( c \)

3. \( \frac{d}{dx}[f(x)g(x)] = f(x) \frac{d}{dx}[g(x)] + g(x) \frac{d}{dx}[f(x)] \)

4. \( \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \), provided \( g(x) \neq 0 \)

5. \( \frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \)

The third rule is called the product rule, the fourth the quotient rule, and the fifth the chain rule. These differentiation rules can be proven using the limit definition (4) of the derivative function.

The differentiation rules above make calculating derivatives much easier. I use them all throughout Everyday Calculus, starting in Chapter 3; see also Appendixes 3–5 (the quotient rule is mentioned explicitly at the start of Appendix 4).
3.2 Applications of Differentiation

Now that we can calculate derivatives easier, let's briefly discuss the variety of applications of differentiation.

- **Geometric applications.** $f'(a)$ is the slope of the line tangent to the graph of $f$ at the point $(a, f(a))$.
- **Applications to rates of change.** $f'(a)$ is the instantaneous rate of change of $f$ at $x = a$. This is a more physical interpretation of the derivative that can help describe how real-world quantities (e.g., velocity, temperature, etc.) change from instant to instant. See Chapter 2 and onward of *Everyday Calculus* for many examples, including applications to biology, economics, and physics.
- **Linearization.** $f'(a)$ is the approximate change in $y$-value of $f$ after a one-unit increase in $x$ from $a$. This is called the linearization interpretation of the derivative. It has a variety of applications; see equation (15) and the ensuing discussion in *Everyday Calculus*.
- **Related rates.** If we encounter an equation that relates two or more variables which each change, say in time, then often we can differentiate the equation to relate the variable’s instantaneous rates of change (i.e., their derivatives). Such situations lead to “related rates” problems. Figure 4.2 in *Everyday Calculus* discusses one such related rates problem: the relationship between how fast the volume of liquid in a cup is increasing to the liquid’s water level inside the cup.
- **Differentials.** Because derivatives help quantify changes in a function, they can be used to estimate percent changes in outputs of the function given percent changes in inputs of the function; this is typically done with “differentials.” The first subsection of Chapter 5 in *Everyday Calculus* illustrates this through an application to blood flow during exercise.
- **Optimization.** This is perhaps the most important application of derivatives. Nearly all of Chapter 5 in *Everyday Calculus* is devoted to optimization applications, including to problems in physics, biology, economics, and everyday life (like commute times to work).

As we can see, there are lots of applications of derivatives, owing to the many different interpretations of the concept discovered over the centuries (slope of the tangent line, instantaneous rate of change, etc.). And the list above is not exhaustive; it merely includes the typical applications of differentiation encountered in a first-semester college-level calculus course. Differentiation forms the basis of many other subfields of math (for example, differential equations) that have many, many more applications.

4 Integration

Archimedes—one of the earliest people to stumble upon the concept of what would later become integration—spent a considerable amount of his life working on a problem just as challenging as the “slope of the function” one discussed at the start of the previous section. Archimedes was trying to find the area of various irregularly shaped regions (i.e., not rectangles or triangles). He devised an ingenious approach to solve this problem that came be known as the method of exhaustion.

Archimedes envisioned chopping up the region into smaller pieces that were more regular (say, rectangular)—so that those areas could be calculated—and then adding up the resulting areas to approximate (and, ultimately, calculate) the area of the original region. Archimedes used his method to successfully calculate the correct area of a circle of radius $r$ ($\pi r^2$), quite the accomplishment at the time (circa 260 B.C.). Over the centuries that followed, mathematicians further developed Archimedes’ results and honed in on solving the following problem: Given a function $f$, find the
area of the region bounded by the graph of $f$, the $x$-axis, and $a \leq x \leq b$ (for example, the shaded region in Figure 3). This came to be called “the area problem.”

![Figure 3: The graph of $f(x) = x^2$ with the area bounded by it, the $x$-axis, and $0 \leq x \leq 1$ shaded.](image)

Solving the area problem ultimately yielded what we today called **integral calculus**. To illustrate the main idea, let’s first approximate the shaded region in Figure 3. The easiest way to do this is to inscribe rectangles in the Region. Let’s now work out what those rectangles’ widths and heights will be.

**The rectangles’ widths.** To keep things easy, let’s make the rectangles have equal width $\Delta x$. Since the entire interval width is 1 (since $0 \leq x \leq 1$), if we want to inscribe $n$ rectangles of equal width $\Delta x$, then

$$n\Delta x = 1 \implies \Delta x = \frac{1}{n}.$$ 

For example, Figures 4(a)–(b) illustrate the cases of $n = 2$ and $n = 4$, respectively.

![Figure 4: The graph of $f(x) = x^2$ with (a) one inscribed rectangle of width $\Delta x = 0.5$, and (b) three inscribed rectangles of width $\Delta x = 0.25$.](image)

You may notice that although I said the Figure illustrates the $n = 2$ and $n = 4$ cases, there are only 1 and 3 inscribed rectangles, respectively. This is due to the other half of the problem we’ve yet to quantify: the heights of the rectangles.

**The rectangles’ heights.** The rectangles shown in the Figure have heights equal to $f(x)$, where $x$ is the left-endpoint of the $x$-region defining their widths. In both figures, this means the first rectangle’s
height is \( f(0) = 0 \), which is why we don’t see that extra rectangle. To figure out the heights of the other rectangles we need to find the left endpoints of the “partition” created by subdividing \( 0 \leq x \leq 1 \) into \( n \) subintervals. Here’s how we do that.

Since each rectangle has width \( \Delta x \), splitting the interval \( 0 \leq x \leq 1 \) into \( n \) subintervals of length \( \Delta x \) produces the “partition points”

\[
x_0 = 0, \quad x_1 = \Delta x, \quad x_2 = 2\Delta x, \ldots \quad x_n = n\Delta x = 1.
\]

Calling each individual partition point \( x_i \), we get the formula

\[
x_i = i\Delta x = i \left( \frac{1}{n} \right) = \frac{i}{n}, \quad i = 0, 1, \ldots, n.
\]

Thus, the height of each rectangle is:

\[
f(x_i) = f \left( \frac{i}{n} \right).
\]

**Approximating the area of the shaded region.** Now that we know each rectangle’s width and height, we can add up their areas to approximate the area of the shaded region in Figure 3. The sum of the rectangles’ area is

\[
f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_n)\Delta x = \sum_{i=0}^{n} f(x_i)\Delta x
\]

\[
= \sum_{i=0}^{n} f \left( \frac{i}{n} \right) \Delta x
\]

\[
= \frac{1}{n} \sum_{i=0}^{n} f \left( \frac{i}{n} \right)
\]

The new symbol appearing on the right-hand sides of the equations is the capital Greek letter sigma. In math it denotes the operation of adding whatever is next to the \( \sum \) symbol as \( i \) ranges from the number underneath the symbol to the number on top of it. The final equation, (5), is a particular type of **Riemann sum**; see the discussion right after equation (58) in *Everyday Calculus* for a little history concerning Riemann sums.

**Calculating the area of the shaded region.** As Figures 4(a)–(b) suggest, inscribing more and more rectangles likely yields a more and more accurate estimate of the area. Therefore, it makes sense to define the area of the region in Figure 3 to be

\[
\lim_{n \to \infty} \sum_{i=0}^{n} f(x_i)\Delta x
\]

(the limit of the sum of areas of inscribed rectangles as the number of such rectangles approaches infinity). We denote the result (if it exists) by

\[
\int_{a}^{b} f(x) \, dx
\]

and call it the **definite integral**. This definition matches equation (61) in *Everyday Calculus*, which is the ultimate result of the very similar process described therein. (I won’t put what we’ve done above...
into a “definition box” like I did with Definition 3 of the derivative because the actual definition of the definite integral allows for more complicated partitions, not just the equal width, left-endpoint one we just worked with.) One last quick, but important, note: the definite integral actually calculates net area, the area above the x-axis minus the area below it. That means you could end up getting a negative number for the definite integral. (This would happen, for example, if \( f(x) = -1 \).)

As we now see, the integral—the last pillar of Calculus I mentioned on the first page of this document—is also defined in terms of a limit, like the derivative was. And just like what happened in Example 2—where we calculated \( f'(x) \) directly from its limit definition—calculating the definite integral using the limit approach above is tedious. Luckily, Gottfried Leibniz—the co-inventor of calculus—related integral calculus to differential calculus via what we today call the Fundamental Theorem of Calculus (the name underscores its importance):

**Theorem 2: The Fundamental Theorem of Calculus.** Suppose \( f \) is continuous on the closed interval \([a, b]\), and that there exists another function \( F \) such that \( F'(x) = f(x) \) on \([a, b]\) (\( F \) is called the antiderivative of \( f \) on \([a, b]\)). Then:

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a).
\]

(See equation (68) in *Everyday Calculus* and the surrounding discussion for information on how the Theorem is proven.)

This theorem tells us, among other things, that to calculate the net area of the region bounded by the graph of \( f \), the x-axis, and \( a \leq x \leq b \), we need only calculate \( F(b) - F(a) \). No limit calculations needed! We do, however, need to know \( F \), i.e., we need to find an antiderivative of \( f \). Therefore, this theorem reduces the problem of calculating the definite integral to finding “antiderivatives.” This then leads to a search for the integration techniques that can quickly produce these antiderivatives. Appendix 6 in *Everyday Calculus* discusses a couple of integration techniques.

Let me now show you another side of the Fundamental Theorem of Calculus. Since \( F'(x) = f(x) \), we can rewrite the Theorem’s conclusion as

\[
\int_{a}^{b} F'(x) \, dx = F(b) - F(a).
\]

We can loosely interpret this as saying that the integral of the derivative of \( F \) is \( F \). (There is another half to the Theorem that makes this loose statement more precise.) Indeed, this is the main reason the Theorem has the name it has: it teaches us that integration and differentiation are inverse processes, that is, one undoes the other.

That last insight is extremely important, especially given all the applications of differentiation I highlighted in Section 3.2 of this document. I spent most of Chapters 6 and 7 in *Everyday Calculus* discussing a few of the highlights of the applications of integration, including helping us estimate the age of the Universe.
5 Conclusion

I hope you’ve enjoyed this short summary of the mathematics of calculus. There is certainly more to be said, and if you’ve yet to take a formal course on calculus, I highly recommend you do. You’ll likely find that this document—as well as Everyday Calculus—will have already provided you with a working knowledge of calculus and its applications.

Hoping you continue studying math,

Oscar E. Fernandez